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DIAGNOSTIC MEASURES FOR COMPARING DIRECT AND AGGREGATIVE SEASONAL ADJUSTMENTS

Jeff Cannon

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INQUIRIES

- *For further information about the contents of this Working Paper, please contact the author:* Jeff Cannon — *telephone:* Canberra 02 6252 7137 *or facsimile:* Canberra 02 6252 8015 *or e-mail:* jeff.cannon@abs.gov.au

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Abstract

In collections of related time series data, several component series are often added together to produce a total series. Seasonal adjustment at the total level may be done by directly adjusting the total series. An alternative is to directly seasonally adjust the component series and obtain an estimate of the adjusted total by adding together the adjusted components. The latter approach is often described as an aggregative adjustment. A direct and an aggregative seasonal adjustment of the total series will generally produce non-identical results. This paper describes various diagnostic measures which can be used to help determine which approach gives the best results for particular series.

Keywords: Seasonal adjustment, aggregation

1 Introduction

When a collection of series is to be seasonally adjusted for the first time or when additional series are to be added to an existing collection, a decision must be made on the most appropriate level at which to perform the seasonal adjustment of component series. Often there is a degree of flexibility in selecting the level of aggregation, and there are various factors that need to be taken into account when deciding on the best level for adjustment purposes. User demand for seasonally adjusted estimates of particular components is one important consideration. It is also important to determine whether the proposed adjustments will be of a sufficient quality.

Many times when a set of time series requires seasonal adjustment, the set will include subtotal and total series consisting of the sums of other series. These series can either be directly seasonally adjusted in their own right, or alternatively seasonally adjusted estimates can be found by adding up the seasonally adjusted components. The first method is called a direct adjustment and the second method is called an aggregative adjustment. The purpose of this working paper is to discuss various diagnostic measures that can be used to quantify different aspects of seasonal adjustment quality in the context of examining the relative merits of direct versus aggregative seasonal adjustments for particular series.

2 Why direct and aggregative adjustments produce different estimates

Directly adjusting a set of component series and then adding the seasonally adjusted estimates to form an aggregative total can result in a somewhat different seasonally adjusted estimate at the total level than a direct adjustment at the total level would produce. This can occur if the individual components have widely differing seasonal patterns and as a result of nonlinearities in the seasonal adjustment process arising from the modification for extremes and multiplicative manipulations.

The modification for extremes algorithm which forms part of the X11 seasonal adjustment process is the main contributor to the differences which occur. Sometimes the seasonal-irregular value for a given month in a particular year is an outlier or 'extreme' observation which takes on a very different value from other values in that month. If such values are processed without being modified in some way, a seriously distorted estimate of the seasonality for that month can result. To avoid this, seasonal-irregulars that are identified as extreme are replaced with a weighted average of nearby values which are regarded as being more typical. Extreme seasonal-irregular values of opposite sign in component series can cancel each other out when added together, leading to a different interpretation of which values are to be treated as extreme at the more aggregated level.

Another possible source of discrepancies arises from the parameter settings that are employed for the adjustment of the different series. Such parameter settings include the choice of which seasonal and trend moving averages are to be used, and the settings are normally chosen separately for each series so as to optimise the individual adjustments. It can easily happen that various component series use different parameter settings for their adjustment.

For example, if two component series are individually seasonally adjusted, one component may be adjusted using a 3*5 seasonal moving average while the other component is adjusted using a 3*9 seasonal moving average. If the two components are added together to form a total and then seasonally adjusted, whichever seasonal moving average is used on the total must differ from the seasonal moving average applied to at least one of the components. The settings chosen for other adjustment parameters such as the trend moving average can cause similar problems.

3 Some issues to consider when choosing between direct or aggregative adjustments

It is preferable to avoid producing a set of seasonally adjusted estimates that are inconsistent with each other. If seasonally adjusted estimates of the component series are definitely required then the total series is often obtained by aggregation to ensure consistency. An exception to this practice is the seasonal adjustment of Retail Trade series where the Australia Total and various component series are all directly adjusted. A forcing procedure is used to distribute the discrepancy between the direct and aggegative adjustments across the components in a pro-rata fashion. However the forcing process introduces an additional degree of complexity into the periodic updating of seasonally adjusted estimates as additional data becomes available and so is not widely used.

Often however the quality of adjustment of higher level subtotal and total series assumes paramount importance and the quality of estimates at finer levels of disaggregation is a secondary consideration. For example, in many collections interest centers on the 'headline' Australia Total series, with a smaller interest in State Totals, and less again on finer level splits within States.

The quality of the adjustments tends to decline as the series are broken down into finer level components, and if the seasonal adjustment of disaggregated splits is attempted at too low a level the results may be so unreliable as to be of little use. The reason for this is that the seasonal pattern which is estimated and then removed by the seasonal adjustment process may be regarded as a signal which is mixed with an irregular or noise element. The noise element consists of a series of random fluctuations which have a tendency to cancel to a degree when component series are added together. More aggregated series usually have a better signal-to-noise characteristic than the contributing component series, which enables a more accurate estimate of the seasonality to be made.

When deciding on which seasonal adjustment method to use, it is useful to have a set of tests or measures that can be used to compare the adjustment quality of aggregative versus direct adjustment for particular series. Tests developed by various time series researchers and reported in the literature focus on three different aspects of seasonal adjustment quality. These aspects are:

1) tests for residual seasonality

2) smoothness measures, and

3) measures of adjustment stability.

The purpose of seasonal adjustment is to remove the seasonal variation from the seasonally adjusted series. If there is still some seasonal pattern remaining in the seasonally adjusted series then this is evidence of a poor quality adjustment.

If an aggregative adjustment of a series displays residual seasonality but a direct adjustment does not, then there are grounds for preferring the direct adjustment over the aggregative adjustment. Similarly if a direct adjustment displays residual seasonality and an aggregative adjustment does not then the direct adjustment would be considered to be superior. If neither the direct nor the aggregative adjustment displays residual seasonality then both adjustments would be considered satisfactory with regard to the residual seasonality criterion. In the unfortunate circumstance that both the aggregative and direct adjustments display residual seasonality then the method with the smaller amount of residual seasonality would be considered the better method.

Smoothness measures are based on the premise that users do not like large period-to-period movements in the seasonally adjusted series as it makes the data more difficult to interpret, so the smoother the seasonally adjusted series the better. If an aggregative adjustment produces a smoother seasonally adjusted series than the corresponding direct adjustment then, all other things being equal, the aggregative adjustment would be considered superior. If the direct adjustment gives a smoother result then this favours the use of direct adjustment.

Measures of adjustment stability are an attempt to quantify the problem of revisions. Seasonally adjusted estimates that undergo large revisions when they are recalculated as additional time series values become available may cause users to lose confidence in the usefulness of the adjusted data. If the size of revisions is generally smaller for an aggregative adjustment than for a direct adjustment then the aggregative adjustment is preferred, if the size of revisions is smaller for a direct adjustment then the direct adjustment is preferred.

Diagnostic measures of the three aspects of seasonal adjustment quality are discussed in more detail in the following sections.

4 Residual seasonality

Direct and aggregative adjustments of the same original data will, in general, produce non-identical seasonally adjusted series. This implies that somewhat different seasonal cycles in the original data have been identified and removed. If not all of the seasonal cycles are removed by the process of seasonal adjustment then the seasonally adjusted series will contain residual seasonality. The presence of residual seasonality is evidence of a poor quality adjustment.

In the case of direct adjustments there are tests for residual seasonality (printed at the foot of the D11 table in X11) which are intended to guard against residual seasonality in the seasonally adjusted series. Given that an aggregatively adjusted series contains some different cycles from the direct adjustment, the aggregatively adjusted series should also be tested for the presence of residual seasonality. There are three different approaches to testing for residual seasonality that will be discussed further. These are the use of F tests, spectral methods, and readjusting seasonally adjusted series to detect residual seasonality.

4.1 F tests

The X11 seasonal adjustment program includes various F tests which are used to test for the presence of stable seasonality (tables B1 and D8), moving seasonality (table D8), and residual seasonality (table D11). All of these tests could potentially be used to test for residual seasonality in an aggregative seasonally adjusted series. In the normal use of X11 the tests (except for the test in table D11) are designed to test whether there is enough seasonality in the original series for it to be worthwhile carrying out an adjustment. If the input series is a seasonally adjusted series then the F tests provide an indication whether there is any remaining seasonality left in the input adjusted series.

The existing tests for residual seasonality in X11 (at table D11) were originally added by Statistics Canada in their X11-ARIMA version of X11. They check for residual seasonality over the entire series and over the last 3 years of the data span. Dagum (1980[1]) outlined that the tests are applied to the seasonally adjusted series from which the trend has been removed by first-order differencing at lag 3 for monthly series and lag 1 for quarterly series. For series that display a multiplicative relationship between the trend and residual-irregular elements, first-order differencing would result in the tests being applied to a residual-irregular series with non-stationary variance. A more appropriate procedure might be to divide the seasonally adjusted series by the trend to produce a multiplicative residual-irregular series with near constant variance. Unfortunately a copy of Higginson (1976[2]) which may shed more light on the reasons for the procedure adopted in X11-ARIMA could not be located.

Once having obtained a residual-irregular series, comprehensive testing for residual seasonality would involve statistical tests for the presence of both moving and stable seasonality, possibly over a shorter subspan of data as well as over the entire series. In the subspan case using fewer data points would reduce the power of the tests.

The reason that both stable and moving seasonality should be tested for is that an F test designed only to test for stable seasonality may easily fail to detect the presence of seasonality if the seasonality present is moving. An F test designed to test for moving seasonality involves a two-way analysis of variance testing for differences between years and differences between months or quarters. An F test for stable seasonality is a one-way analysis of variance testing for differences between months or quarters only.

The two-way analysis of variance test must allow for the fact that seasonal factors are adjusted to sum to approximately 12 (monthly) or 4 (quarterly) over any 12 month span. Unless this constraint is broken the test for differences between years will always turn out to be non-significant. The problem is solved by taking the absolute values of the mean corrected seasonal-irregular values. The one-way analysis of variance is not constrained and is based on a different procedure. This means that the F test for stable seasonality would be more sensitive than the test for moving seasonality. Appendix 1 contains details of F tests for stable and moving seasonality, or see Higginson (1975[3] & 1976[4]) and Shiskin et al (1967[5], appendix A) for further details of the tests.

4.2 Spectral analysis

The latest version of the seasonal adjustment program developed by the U.S. Bureau of the Census, X-12-ARIMA, includes two spectral estimation routines. It is envisaged that spectral estimation will be included as a future enhancement to the ABS seasonal adjustment program, which is called Seasabs. There are various different methods that can be used to estimate the spectrum of a series. The two methods employed in X-12-ARIMA are the periodogram and an autoregressive spectrum estimator.

It has been pointed out by authors such as Chatfield (1980[6] p137, 2nd ed) that the periodogram is not a consistent estimator of the spectrum although it is still a useful diagnostic tool. Consistent spectral estimation procedures include smoothing the periodogram, using a spectrum estimator involving a truncation and weighting procedure such as the Tukey or Parzen lag windows, autoregressive spectrum estimation techniques or more recently methods based on time-averaged wavelet analysis have been suggested.

Checking for residual seasonality using a spectral plot consists of estimating the spectrum of the seasonally adjusted series and analysing the plot to see whether there are peaks in the spectrum at seasonal frequencies. The X-12-ARIMA program has a facility that automatically flags series which display possible spectral peaks at seasonal and/or trading day frequencies. The facility is described by Findlay et al (1998[7]):

'Whenever seasonal adjustment is done (with or without trading day adjustment), X-12-ARIMA automatically estimates two spectra, (1) the spectrum of the month-to-month differences of the adjusted series modified for extreme values from X-11 output table E2 (or of the first differences of logarithms of this series with a multiplicative adjustment) and (2) the spectrum of the final irregular component adjusted for extreme values, from output table E3. First differencing is a crude detrending procedure that is usually adequate to enable the spectrum estimate to reveal significant seasonal and trading-day effects. The program compares the spectral amplitude at the seasonal and trading day frequencies .... with the amplitudes at the next lower and higher frequencies plotted. If these neighbouring amplitudes are smaller by a margin that depends on the range of all spectrum amplitudes, then plots of the estimated spectra are automatically printed, together with a warning message that gives the number of "visually significant" peaks found at seasonal or trading day frequencies.'

Using the spectrum to check for residual seasonality may not be an especially powerful technique compared with some of the alternative methods. There are several different seasonal frequencies to check (six seasonal frequencies for monthly series and two for quarterly) and the method essentially relies on the analyst's judgement as to what constitutes a significant peak in the spectrum rather than simply the chance effect of noise.

4.3 Readjusting the seasonally adjusted series

The third method of checking for residual seasonality, and the one which has been used most often in ABS practice to date, is to take an aggregative seasonally adjusted series as the input series and run it through Seasabs a second time. As part of the resulting attempted adjustment the F tests in X11 which check for stable, moving and identifiable seasonality are applied, and an 'SI' chart which shows the detrended input series grouped by month or quarter is produced.

The expert system built into Seasabs can weigh up the various diagnostic outputs from X11 and produce an assessment as to whether significant seasonality is present in the input series. If the conclusion is that the series can be successfully seasonally adjusted, then by implication the input series must contain an identifiable seasonal element. If on the other hand the diagnosis is that the series should not be seasonally adjusted because seasonality is not present, then this can be taken as an indication that the input series does not contain significant residual seasonality. Since the expert system diagnosis relies on decision rules built into the program it gives a consistent measure that does not require the analyst to make judgements on a series-by-series basis.

A further aid to detecting residual seasonality in a seasonally adjusted time series is the 'SI' chart, which shows the clustering of detrended input values in relation to the neutral line, grouped by month or quarter. An 'SI' chart with one or more groups clustered above or below the neutral line is evidence of residual seasonality. The 'SI' chart requires interpretation by the analyst and is therefore a subjective measure, however the results may be easier to interpret than a spectrum plot, particularly for time series analysts who are likely to have considerable experience in interpreting SI charts.

5 Smoothness measures

The use of numerical measures of the smoothness of the seasonally adjusted series as an indicator of adjustment quality is based on the premise that users don't like large period-to-period movements in seasonally adjusted data, so the smoother the seasonally adjusted series the better.

It could be argued that the degree of smoothness is not really an appropriate indicator of seasonal adjustment quality since the purpose of seasonal adjustment is not to produce a smooth series as such. Rather the purpose is to remove seasonal variation from the original series, leaving a series that contains both trend and residual-irregular variation. Some critics (eg Nettheim 1965[8]) of the X11 seasonal adjustment process have claimed that the method is prone to over-adjustment: the removal of more variation at seasonal and near-seasonal frequencies than is warranted. If such criticisms are valid, then a smoother seasonally adjusted series could be the result of over-adjustment rather than a better quality adjustment.

Those users who require a smooth series can use the trend estimate instead of the seasonally adjusted series. Commentary published by the ABS has for several years emphasised that for many purposes the trend estimate provides a better guide than the seasonally adjusted series.

Despite ABS attempts to encourage a greater emphasis on trend estimates, the focus of media commentary continues to rest largely on seasonally adjusted data, often with an emphasis on the most recent movement in the seasonally adjusted series. ABS experience has shown that large period-to-period movements in seasonally adjusted series can prompt an excited reaction from the user community so from a practical perspective an adjustment that provides a relatively smooth seasonally adjusted series is desirable.

5.1 R measures

The X11-ARIMA program developed by Statistics Canada includes two measures intended to distinguish whether an aggregative or a direct seasonal adjustment gives superior results. The two measures, designated R_1 and R_2 (Dagum 1980[1], Chapter 2), measure the roughness, or lack of smoothness, of the seasonally adjusted series. The adjustment which gives smaller R measures is considered the better adjustment.

Findlay et al (1990[9]) noted that 'there are no theoretical models of seasonality whose ideal seasonal adjustment minimises a quantity estimated by R_1 or R_2. For this reason, the use of such measures to compare adjustments (smoother is better) is somewhat unsatisfactory '.

Despite the lack of theoretical underpinning, the R measures provide a useful guide to the typical size of movements in the seasonally adjusted series. User preference for smaller period-to-period movements in the seasonally adjusted series can be accommodated by selecting either the direct or aggregative adjustment depending on which has the smaller R measures.

The measures are described by Findlay et al (1990[9]) as follows:

R_1 is the mean of squares of first differences of the seasonally adjusted series:

$$R_1 = (N - 1)^{-1} \sum_{t=2}^{N} (A_t - A_{t-1})^2$$

R_2 is the mean of squares of additive residual-irregulars (seasonally adjusted minus trend):

$$R_2 = N^{-1} \sum_{t=1}^{N} (A_t - H_t)^2$$

where A_t ($t = 1, .. , N$) is the seasonally adjusted series (direct or aggregative), H_t ($t = 1, .. , N$) is the associated trend obtained by smoothing the seasonally adjusted series with Henderson trend weights, and N is the length of the series.

The two R measures are similar to each other, but since the seasonally adjusted series consists of trend and residual-irregular components the R_1 measure can be adversely affected if the series contains strong short-term trend cycles. In these circumstances it is preferable to use the R_2 measure.

In X11-ARIMA the direct and aggregative adjustments are compared using percentage difference values, defined as

$$\Delta_i = 100 * (R_i^{direct} - R_i^{aggregative}) / R_i^{direct}, \; i = 1 \text{ or } 2,$$

so that negative values of Δ_1 or Δ_2 favour direct adjustment.

5.2 Other measures

There are various other similar measures that could plausibly be used to measure the roughness of the seasonally adjusted series such as the average absolute percentage change period-to-period in the seasonally adjusted series (AAPC(SA)), or the mean of squares of multiplicative residual-irregulars around 1.0 (MSI). Formally, these measures may be defined as:

$$AAPC(SA) = (N - 1)^{-1} \sum_{t=2}^{N} |100 * (A_t - A_{t-1}) / A_{t-1}|$$

$$MSI = N^{-1} \sum_{t=1}^{N} (I_t - 1.0)^2$$

where $I_t = A_t / H_t$

These measures are open to criticism on the same theoretical grounds as the R_1 and R_2 measures, but may be more appropriate for multiplicative series than the R measures since they weight the 'roughness' more evenly as the level of the series rises and falls over time.

6 Measures of adjustment stability

It is an inevitable consequence of the seasonal adjustment process that seasonally adjusted estimates are revised as additional data becomes available, either annually in the case of forward factor adjustments or each month or quarter in the case of concurrent adjustments. Seasonally adjusted estimates that undergo large revisions when they are recalculated as future time series values become available may cause users to lose confidence in the usefulness of the adjusted data.

It is desirable that the revisions to the seasonally adjusted estimates be as small as possible at each update, and that the estimates converge quickly to their final values rather than undergoing a sequence of revisions that continues for many years as subsequent data is incorporated into the analysis. Therefore it is useful to have numeric measures which quantify the extent of revisions. When comparing direct and aggregative adjustments these measures allow identification of the method which results in smaller revisions (ie more stable adjustments).

6.1 Star measures

One such numeric measure currently used within the ABS is the 'star measure'. The value of the star measure for directly multiplicatively adjusted series is equal to the average absolute percentage change period-to-period in the residual-irregular component, or AAPC(I). An empirical rule of thumb linking the value of the star measure to the expected degree of revision of the seasonally adjusted series states that 'generally, the average percentage revision (without regard to sign) for the most recent year, will be approximately one half of the series' star value. This degree of revision can be expected to fall gradually to about one fifth of the star value for years four or more earlier'.

The empirical rule of thumb has been developed with reference to directly adjusted series and may provide a less accurate guide to the stability of aggregative adjustments. Further study is required in order to resolve this question.

For directly adjusted series an estimate of the residual-irregular component is available from the D13 table of X11. This table is not available for aggregatively adjusted series since they are summed from component series rather than being adjusted using X11. An alternative estimate of the residual-irregular component that can be made for both direct and aggregative adjustments is to divide the seasonally adjusted series by the trend estimate to obtain a multiplicative residual-irregular series.

This procedure may not work successfully for additive adjustments. If the trend series changes sign then division by zero, or by a quantity very close to zero, may occur. Star measures are not presently calculated for direct additive adjustments. Since the vast majority of time series adjusted by the ABS are adjusted multiplicatively, the star measure has proven to be a useful guide to adjustment stability in practice.

6.2 Sliding spans

Findlay et al (1998[7]) discuss two types of stability diagnostics that are included in the X-12-ARIMA program, namely sliding spans and revision histories. Sliding span diagnostics are also discussed in more detail in Findlay et al (1990[9]). The sliding span diagnostics compare different outcomes obtained by running a seasonal adjustment on up to four overlapping subspans of the series. For each period that is common to at least two of the subspans, these diagnostics analyse the difference between the largest and smallest seasonal adjustment factors for that period obtained from the different spans. They also analyse the largest and smallest estimates of period-to-period changes in the seasonally adjusted series and calculate analogous trend measures.

The length of span that is used depends on the seasonal filter used for direct adjustment:

Seasonal filter	Length of span
3 * 3	6 years
3 * 5	8 years
3 * 9	11 years

The following diagram illustrates four 8 year spans of a series starting in January 1988 and ending in December 1998. January 1994 falls within all four spans so separate adjustment of each span will result in four candidate seasonal adjustment factors $S(1)_{1,94}$, $S(2)_{1,94}$, $S(3)_{1,94}$ and $S(4)_{1,94}$ for this month.

```
Y1,88 ----------------- 1st span
.                  | 1/88 to 12/95
.                  |
Y12,88             |
Y1,89              |------------------------ 2nd span
.                  |                  | 1/89 to 12/96
.                  |                  |
Y12,89             |                  |
Y1,90              |                  |------------------------ 3rd span
.                  |                  |                  | 1/90 to 12/97
.                  |                  |                  |
Y12,90             |                  |                  |
Y1,91              |                  |                  |------------------------ 4th span
.                  |                  |                  |                  | 1/91to12/98
.                  |                  |                  |                  |
Y12,91             |                  |                  |                  |
Y1,92              |                  |                  |                  |
.                  |                  |                  |                  |
.                  |                  |                  |                  |
Y12,92             |                  |                  |                  |
Y1,93              |                  |                  |                  |
.                  |                  |                  |                  |
.                  |                  |                  |                  |
Y12,93             |                  |                  |                  |
Y1,94    S(1)1,94  |     S(2)1,94     |     S(3)1,94     |     S(4)1,94     |
.                  |                  |                  |                  |
.                  |                  |                  |                  |
Y12,94             |                  |                  |                  |
Y1,95              |                  |                  |                  |
.                  |                  |                  |                  |
.                  |                  |                  |                  |
Y12,95--------------                  |                  |                  |
Y1,96                                 |                  |                  |
.                                     |                  |                  |
.                                     |                  |                  |
Y12,96---------------------------------                  |                  |
Y1,97                                                    |                  |
.                                                        |                  |
.                                                        |                  |
Y12,97----------------------------------------------------                  |
Y1,98                                                                       |
.                                                                           |
.                                                                           |
Y12,98 ----------------------------------------------------------------------
```

Each month that falls within two or more of the spans will have at least two different estimated seasonal adjustment factors. The extent of differences between the seasonal adjustment factor estimates from different spans gives a measure of the adjustment stability.

Let $S(k)_t$ denote the seasonal adjustment factor from span k for month t. For each month that falls within two or more spans, calculate the maximum difference between the estimated seasonal adjustment factors:

$$S_t^{max} = (\max S(k)_t - \min S(k)_t) / \min S(k)_t$$

Findlay suggests that the seasonal adjustment factor estimate is unreliable if $S_t^{max} > 0.03$, ie if the difference between the highest and lowest value of the estimated seasonal adjustment factor is greater than 3 percent. In order to obtain a summary measure of the adjustment stability, the percentage of months with $S_t^{max} > 0.03$ is then calculated, and designated S(%). The higher the value of S(%), the more unstable the seasonal adjustment is considered to be.

This measure can be used to make comparisons between direct and aggregative adjustments, since implicit seasonal adjustment factors may be obtained for aggregative adjustments by dividing the original series by the corresponding seasonally adjusted series. However it would be necessary to carry out aggregative adjustments over different spans in order to calculate the implicit $S(k)_t$ and this may cause some operational difficulties in practice.

The use of sliding-span diagnostics is not confined to comparisons between direct and aggregative adjustments. Findlay describes other diagnostic measures such as the month-to-month and year-to-year percentage change in the adjusted value from span k for month t. These can test the quality of direct seasonal adjustments and the reliability of trading day estimates, and aid the selection of appropriate seasonal filters.

6.3 Revisions histories

The second type of stability diagnostic in X-12-ARIMA considers the revisions associated with continuous seasonal adjustment over a period of years, and is referred to as a revisions history. It is described (in 1998[7]) in the following terms: 'The basic revision calculated by the program is the difference between the earliest adjustment of a month's datum obtained when that month is the final month in the series and a later adjustment based on all future data available at the time of the diagnostic analysis. Similar revisions are obtained for month-to-month changes, trend estimates, and trend changes. Sets of these revisions, calculated over a consecutive set of time points within the series, are called revisions histories.

'Suppose a set of options has been chosen for the application of X-12-ARIMA to the unadjusted time series Y_t, $1 \le t \le N$. For any of these months t and any integer u in the interval $t \le u \le N$, let $A_{t|u}$ denote the seasonally adjusted value for time t obtained with these options when only the data Y_t, $1 \le t \le u$, are used in their calculation (Y_{u+1}, ..., Y_N are withheld). For given t, as u increases these adjustments converge to a final adjusted value. When the 3 * m seasonal filter is used, convergence is is usually effectively reached in about 1 + m / 2 years. The largest revisions tend to occur when u is the same calendar month as t, specifically u = t + 12, t + 24, ... and the next to largest changes a month later, u = t + 1, t + 13, t + 25, ... (In the additive decomposition case, the largest weights in the seasonal adjustment filter combining all of the seasonal adjustment calculations are at lags 1, 12, 13, 24, 25, ...).

'The adjustment $A_{t|t}$ obtained from data through time t is called the *concurrent* adjustment. It is usually the first adjustment obtained for month t. We call $A_{t|N}$ the *most recent* adjustment. In the case of a multiplicative decomposition, the revision from the concurrent to the most recent adjustment for month t is calculated by the program as a percentage of the concurrent adjustment, $R_{t|N}^{A} = 100 * (A_{t|N} - A_{t|t}) / A_{t|t}$. For given N_0 and N_1 with $N_0 < N_1$, the sequence $R_{t|N}^{A}$, $N_0 \leq t \leq N_1$, is called a *revision history* of the seasonal adjustments from time N_0 to N_1. We suggest that N_0 be at least as large as the effective length of the seasonal filter used, 12 * (2 + m). It should definitely be large enough for reliable estimation of any trading-day or holiday adjustments being performed.

'Period-to-period percent changes, $\Delta^{\%}A_{t|u} = 100 * (A_{t|u} - A_{t-1|u}) / A_{t-1|u}$ are often as important as the seasonal adjustments. X-12-ARIMA can produce revision histories for them: $R_{t|N}^{\Delta\%A} = \Delta^{\%}A_{t|N} - \Delta^{\%}A_{t|t}$, $N_0 \leq t \leq N_1$. The program also calculates the analogous quantities for final Henderson trends $T_{t|u}$ and for their period-to-period percent changes $\Delta^{\%}T_{t|u}$. These histories are denoted by $R_{t|N}^{T}$ and $R_{t|N}^{\Delta\%T}$, $N_0 \leq t \leq N_1$.'

Prior to calculating a revisions history, the start and end points of the revisions history span (ie N_0 and N_1) must be determined. The recommendation given by Findlay is that N_0 = 12 * (2 + m), and N_1 = N - 12 * (1 + m / 2). For example we may have a monthly series containing 11 years of data which starts in January 1988 and ends in December 1998. If this series is adjusted using a 3 * 5 seasonal moving average then m = 5. There are 11 * 12 observations ie N = 132 , N_0 = 12 * (2 + 5) = 84 and N_1 = 132 - 12 * (1 + 5 / 2) = 90. This corresponds to starting the revisions history span in December 1994 and ending it in June 1995.

Then the sequence $R_{t|N}^{A}$, $N_0 \leq t \leq N_1$, can be obtained by running seasonal adjustments with end dates ranging from N_0 to N_1 and comparing the concurrent adjustments with the corresponding most recent adjustments (ie comparing initial to 'final' estimates for each time point in the range N_0 to N_1).

As the example given illustrates, a considerable length of data may be required in order to carry out a revisions history if the start and end dates are set according to the recommendations given by Findlay et al. Let N_1' be the number of observations back from the end of the series that the revisions history span ends (ie N_1' = 12 * (1 + m / 2)).

The minimum number of observations N for various choices of seasonal moving average is:

Seasonal moving average		Monthly			Quarterly		
SMA	m	N_0	N_1'	N (minimum)	N_0	N_1'	N (minimum)
3 * 3	3	60	30	90 (7.5 years)	20	10	30 (7.5 years)
3 * 5	5	84	42	126 (10.5 years)	28	14	42 (10.5 years)
3 * 9	9	132	66	198 (16.5 years)	44	22	66 (16.5 years)

The minimum values given will only permit a revisions history for a single month or quarter. In order to be useful the revisions history span should cover several months or quarters which means that for practical purposes the minimum number of observations in a series would need to be somewhat higher than indicated in the table.

Findlay considers both average absolute percent revisions and the number of extreme revisions over the revisions history span. A revision of greater than 4 percent from initial to final of the seasonally adjusted series is considered to be extreme, however this value is clearly somewhat arbitary and would need to be set with reference to the series in question.

The Avg $| R_{t|N}^A |$ and No. $| R_{t|N}^A | > 4.0\%$ measures could be used to compare aggregative and direct adjustments. A smaller value of each measure is indicative of a more stable adjustment. In practice the need to run aggregative adjustments over a range of different end dates may cause some operational difficulties in the current ABS environment.

7 Combining the diagnostics

In order to use the diagnostic measures as a basis for selecting a direct or an aggregative adjustment, the various measures need to be combined to give an overall picture of adjustment quality for each method. How this may best be accomplished is a topic for further research and no definitive recommendations are given here. What follows is a brief outline of one possible approach.

A weighted combination of the various diagnostic measures could be used to generate an overall measure of adjustment quality. For example, if a test for the presence of residual stable seasonality is not significant at the 5% level then a weight (or score) of 0 might be appropriate. If the test is significant at 5% but not at 1% then a weight of say 5 might be given, if the test is significant at 1% a weight of 20 could be given and if the test is significant at 0.1% a weight of 100 might be given.

Smoothness measures such as the Average Absolute Percentage Change period-to-period in the seasonally adjusted series (AAPC(SA)) are in fact measures of the extent to which the series is not smooth, so a smaller value of the statistic indicates a smoother series. A smoothness score could be derived by multiplying the smoothness measure by an appropriate coefficient. The coefficient would need to be chosen so that the characteristic of smoothness is given an appropriate degree of importance in the overall picture of adjustment quality.

Revision measures could be handled in a similar way to the smoothness measures. Since smallness of the degree of revisions is sought, numeric measures of the average size of revisions could be multiplied by an appropriate coefficient to obtain a revisions score. The various scores could then be added together to obtain an overall score for the series, with a smaller score being indicative of a higher quality adjustment.

If the overall adjustment quality for series i is denoted Q_i and the various quality measures are denoted M_{ij} for the different measures 1 to m, then Q_i would be calculated as

$$Q_i = \sum_{j=1}^{m} X_j * M_{ij}$$

, using a suitably chosen set of coefficients X_j.

This suggestion is in the spirit of the existing quality statistic or 'Q stat' that is calculated by X11 as a numeric measure of overall adjustment quality but for direct adjustments only (the Q stat is a weighted sum of 11 individual quality measures).

Overall quality measures of individual series do not on their own provide a complete picture of the overall adjustment quality of a collection of series. An additional consideration is that some of the series may be of more interest or importance to the majority of users than others. Typically it is the main aggregates that are of most interest, such as Australia and State Totals. The quality of adjustment of these series is of primary importance and an adjustment method that ensures a good outcome for the most important series at the expense of lower quality adjustments of less important series seems sensible.

This suggests the use of a weighting function which specifies the relative importance of each series in the collection. The construction of such a weighting function would involve a degree of political sensitivity as a trade-off between the competing needs of different groups of users would be involved. For example, a weighting function that places a lot of importance on Australia level employment estimates and relatively little on employment estimates for the Northern Territory may be reasonable in some sense given that the Northern Territory is one of the smaller territories. The Northern Territory government might however be less than impressed with such a scheme.

Assuming that a set of weights W_i can be selected which reflect the relative importance of each series in a collection, a global quality measure Q_g can be calculated by appropriately weighting the combined quality measure for each series and summing over all series. If there are n series in the collection then

$$Q_g = \sum_{i=1}^{n} W_i * Q_i.$$

Finding the best method of adjustment for the collection as a whole would then require systematically calculating Q_g for each possible combination of aggregative and direct adjustments, and selecting the adjustment which minimises the global measure (since a smaller score is indicative of a higher quality adjustment).

8 Seasabs enhancements

At the time of writing additional functionality is being added to Seasabs as part of an ongoing development program. The additional functionality is primarily intended to facilitate the introduction of concurrent seasonal adjustment to one or more additional ABS collections, but some of the new diagnostics are also applicable to the selection of aggregative or direct seasonal adjustments. In addition, other diagnostic measures intended for comparing aggrgative and direct adjustments are being incorporated. As a result of the current round of enhancements it is expected that in future Seasabs will include the following functionality:

1) F test for residual stable seasonality
See Appendix 1 for details of the test.

2) F test for residual moving seasonality
See Appendix 1 for details of the test.

3) R_1

R_1 is the mean of squares of first differences of the seasonally adjusted series:

$$R_1 = (N - 1)^{-1} \sum_{t=2}^{N} (A_t - A_{t-1})^2$$

where A_t (t = 1, .. , N) is the publication seasonally adjusted series and N is the length of the series.

4) R_2

R_2 is the mean of squares of additive residual-irregulars (seasonally adjusted minus trend):

$$R_2 = N^{-1} \sum_{t=1}^{N} (A_t - H_t)^2$$

where A_t (t = 1, .. , N) is the publication seasonally adjusted series , H_t (t = 1, .. , N) is the publication trend series and N is the length of the series. Note that this measure is calculated the same way for both multiplicative and additive adjustments.

5) AAPC(S.A.) or AAC(S.A.)

For multiplicative adjustments the average absolute percentage change period-to-period in the publication seasonally adjusted series, for additive adjustments the average absolute change period-to-period in the publication seasonally adjusted series.

$$AAPC(SA) = (N - 1)^{-1} \sum_{t=2}^{N} |100 * (A_t - A_{t-1}) / A_{t-1}|$$

$$AAC(SA) = (N - 1)^{-1} \sum_{t=2}^{N} |(A_t - A_{t-1})|$$

where A_t (t = 1, .. , N) is the publication seasonally adjusted series and N is the length of the series.

6) MSI

Mean of squares of irregular series.

Calculate an irregular series by dividing the publication seasonally adjusted series by the publication trend (multiplicative adj.) or by subtracting the publication trend from the publication seasonally adjusted (additive adj.). In the case of multiplicative series subtract 1.0 from the irregulars.

Multiplicative:

$$MSI = N^{-1} \sum_{t=1}^{N} (I_t - 1.0)^2$$

where $I_t = A_t / H_t$

Additive:

$$MSI = N^{-1} \sum_{t=1}^{N} I_t^2$$

where $I_t = A_t - H_t$

7) STAR measures

For multiplicative adjustments the average absolute percentage change period-to-period in the multiplicative irregular series, for additive adjustments the average absolute change period-to-period in the additive irregular series.

Calculate an irregular series by dividing the publication seasonally adjusted series by the publication trend (multiplicative adj.) or by subtracting the publication trend from the publication seasonally adjusted (additive adj.).

Multiplicative:

$$STAR = AAPC(I) = (N - 1)^{-1} \sum_{t=2}^{N} |100 * (I_t - I_{t-1}) / I_{t-1}|$$

where $I_t = A_t / H_t$

Additive:

$$STAR = AAC(I) = (N - 1)^{-1} \sum_{t=2}^{N} |(I_t - I_{t-1})|$$

where $I_t = A_t - H_t$

8) Average absolute percent revision in the seasonally adjusted series

Calculate for both direct and aggregate adjustments. User specifies N_0 and N_1, the start and end dates of the simulation span.

Calculate $R_{t|N}^{A} = 100 * (A_{t|t} - A_{t|N}) / A_{t|N}$ for $N_0 \leq t \leq N_1$

Calculate Avg $| R_{t|N}^{A} |$

9) Average absolute percent revision in the period-to-period movements of the seasonally adjusted series

Calculate for both direct and aggregate adjustments. User specifies N_0 and N_1, the start and end dates of the simulation span.

Calculate $R_{t|N}^{\Delta\%A} = \Delta^{\%}A_{t|N} - \Delta^{\%}A_{t|t}$ for $N_0 \leq t \leq N_1$,

where $\Delta^{\%}A_{t|u} = 100 * (A_{t|u} - A_{t-1|u}) / A_{t-1|u}$

Calculate Avg $| R_{t|N}^{\Delta\%A} |$

9 Summary tables

To conclude this report a summary of the tests and measures for comparing direct and aggregative adjustments that have been discussed is presented in tabular form. In some cases the measures have other uses which may help to justify their inclusion in Seasabs. These other potential uses are indicated.

9.1 Residual seasonality

Test	Operation of test	Other uses of test
F tests		
Test for moving seasonality	1) Calculate aggregative irregular series $I_t = A_t / H_t$ for t = 1 to N 2) Apply F test as set out in Appendix 1 3) If moving seasonality is present the aggregative adjustment is unsatisfactory	
Test for stable seasonality	1) Calculate aggregative irregular series $I_t = A_t / H_t$ for t = 1 to N 2) Apply F test as set out in Appendix 1 3) If stable seasonality is present the aggregative adjustment is unsatisfactory	
Spectral analysis		
Estimate spectrum of seasonally adjusted series	1) Examine spectrum of aggregative seasonally adjusted series for peaks at seasonal frequencies 2) If seasonal peaks are present the aggregative adjustment is unsatisfactory	Can be used to investigate the impact of trading day variation, moving holidays, the business cycle and other cyclical phenomena in a series. It allows examination of the results of different filtering operations, and can provide information on the characteristics of the irregular component.
Readjust seasonally adjusted series		
Expert system diagnosis	1) Save aggregative seasonally adjusted series to a file 2) Seasonally adjust 3) If the adjustment is rated successful the aggregative adjustment is unsatisfactory	

SI charts	1) Save aggregative seasonally adjusted series to a file 2) Seasonally adjust 3) Examine SI charts for evidence of high or low months/quarters 4) If there are high or low months/quarters the aggregative adjustment is unsatisfactory	

9.2 Smoothness measures

Test	Operation of test	Other uses of test
R measures		
R1	1) Calculate $R_1 = (N - 1)^{-1} \sum_{t=2}^{N} (A_t - A_{t-1})^2$ for both aggregative and direct adjustments 2) The adjustment with the lower value of R_1 is rated the better adjustment	
R2	1) Calculate $R_2 = N^{-1} \sum_{t=1}^{N} (A_t - H_t)^2$ for both aggregative and direct adjustments 2) The adjustment with the lower value of R2 is rated the better adjustment	
Other measures		
Average absolute percentage change period-to-period in the seasonally adjusted series	1) Calculate $AAPC(SA) = (N - 1)^{-1} * \sum_{t=2}^{N} \|100 * (A_t - A_{t-1}) / A_{t-1}\|$ for both aggregative and direct adjustments 2) The adjustment with the lower value of AAPC(SA) is rated the better adjustment	
Mean of squares of multiplicative irregulars around 1.0	1) Calculate $MSI = N^{-1} \sum_{t=1}^{N} (I_t - 1.0)^2$ where $I_t = A_t / H_t$ for both aggregative and direct adjustments 2) The adjustment with the lower value of MSI is rated the better adjustment	

9.3 Measures of adjustment stability

Test	**Operation of test**	**Other uses of test**
Star measure		
Average absolute percentage change period-to-period in the irregular series	1) Calculate $AAPC(I) = (N - 1)^{-1} * \sum_{t=2}^{N} \lvert 100 * (I_t - I_{t-1}) / I_{t-1} \rvert$ where $I_t = A_t / H_t$ for both aggregative and direct adjustments 2) The adjustment with the lower value of AAPC(I) is rated the better adjustment	
Sliding spans		
Percentage of periods that fall within at least two spans where the difference between the largest and smallest seasonal adjustment factor is more than 3%	1) Calculate $S_t^{max} = (\max S(k)_t - \min S(k)_t) / \min S(k)_t$ for span k and time t where t falls within at least 2 spans 2) Calculate S(%) = percentage of $S_t^{max} > 0.03$ Perform steps 1) and 2) for both aggregative and direct adjustments 3) The adjustment with the lower value of S(%) is rated the better adjustment	Can be used to test whether seasonal adjustment should be performed on a truncated data span because the adjustment in the earlier years is unreliable Can be used as an aid in choosing the appropriate seasonal moving average to use for direct adjustments
Percentage of periods that fall within at least two spans where the difference between the largest and smallest period-to-period movement in the seasonally adjusted series is more than 3%	1) Calculate $M(k)_t = (A(k)_t - A(k)_{t-1}) / A(k)_{t-1}$ for span k and time t where t and t-1 fall within at least 2 spans 2) Calculate $M_t^{max} = (\max M(k)_t - \min M(k)_t) / \min M(k)_t$ 3) Calculate M(%) = percentage of $M_t^{max} > 0.03$ Perform steps 1), 2) and 3) for both aggregative and direct adjustments 4) The adjustment with the lower value of M(%) is rated the better adjustment	Can be used as an aid in choosing the appropriate seasonal moving average to use for direct adjustments

Percentage of periods that fall within at least two spans where the difference between the largest and smallest year-to-year movement in the seasonally adjusted series is more than 3%	1) Calculate $Y(k)_t$ = $(A(k)_t - A(k)_t-12) / A(k)_{t-12}$ for span k and time t where t and t-12 fall within at least 2 spans 2) Calculate Y_t^{max} = $(\max Y(k)_t - \min Y(k)_t) / \min Y(k)_t$ 3) Calculate $Y(\%)$ = percentage of $Y_t^{max} > 0.03$ Perform steps 1), 2) and 3) for both aggregative and direct adjustments 4) The adjustment with the lower value of Y(%) is rated the better adjustment	Can be used as an indicator that trading day estimates are unreliable Can be used as an aid in choosing the appropriate seasonal moving average to use for direct adjustments
Revisions history		
Average absolute percent revision in the seasonally adjusted series over the history span	1) Calculate $N_0 = 12 * (2 + m)$, $N_1 = N - 12 * (1 + m/2)$ 2) Calculate $R_{t\|N}^A = 100 * (A_{t\|N} - A_{t\|t}) / A_{t\|t}$ for $N_0 \leq t \leq N_1$ 3) Calculate Avg $\| R_{t\|N}^A \|$ Perform steps 2) and 3) for both aggregative and direct adjustments 4) The adjustment with the lower value of Avg $\| R_{t\|N}^A \|$ is rated the better adjustment	The average size of revisions in the seasonally adjusted estimates from their initial to their final stable values may be of interest to users
Number of extreme revisions of more than 4% in the seasonally adjusted series over the history span	1) Calculate $N_0 = 12 * (2 + m)$, $N_1 = N - 12 * (1 + m/2)$ 2) Calculate $R_{t\|N}^A = 100 * (A_{t\|N} - A_{t\|t}) / A_{t\|t}$ for $N_0 \leq t \leq N_1$ 3) Calculate No. $\| R_{t\|N}^A \| > 4.0\%$ Perform steps 2) and 3) for both aggregative and direct adjustments 4) The adjustment with the lower value of No. $\| R_{t\|N}^A \| > 4.0\%$ is rated the better adjustment	

Average absolute percent revision in the period-to-period movements of the seasonally adjusted series over the history span	1) Calculate $N_0 = 12 * (2 + m)$, $N_1 = N - 12 * (1 + m/2)$ 2) Calculate $R_{t\|N}^{\Delta\%A} = \Delta^{\%}A_{t\|N} - \Delta^{\%}A_{t\|t}$ for $N_0 \leq t \leq N_1$ where $\Delta^{\%}A_{t\|u} =$ $100 * (A_{t\|u} - A_{t-1\|u}) / A_{t-1\|u}$ 3) Calculate Avg | $R_{t\|N}^{\Delta\%A}$ | Perform steps 2) and 3) for both aggregative and direct adjustments 4) The adjustment with the lower value of Avg | $R_{t\|N}^{\Delta\%A}$ | is rated the better adjustment	The average size of revisions of the movements in seasonally adjusted estimates from their initial to their final stable values may be of interest to users
Number of extreme revisions of more than 4% in the period-to-period movements of the seasonally adjusted series over the history span	1) Calculate $N_0 = 12 * (2 + m)$, $N_1 = N - 12 * (1 + m/2)$ 2) Calculate $R_{t\|N}\Delta^{\%A} = \Delta^{\%}A_{t\|N} - \Delta^{\%}A_{t\|t}$ for $N_0 \leq t \leq N_1$ where $\Delta^{\%}A_{t\|u} =$ $100 * (A_{t\|u} - A_{t-1\|u}) / A_{t-1\|u}$ 3) Calculate No. | $R_{t\|N}\Delta^{\%A}$ | > 4.0% Perform steps 2) and 3) for both aggregative and direct adjustments 4) The adjustment with the lower value of No. | $R_{t\|N}\Delta^{\%A}$ | > 4.0% is rated the better adjustment	

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Appendix 1

A two-way analysis of variance test for the presence of moving seasonality

The following calculations are for monthly data. Calculations for quarterly data are very similar, the only difference is that in the quarterly case there are 4 periods per year rather than 12.

Let SI_{ij} be the multiplicative seasonal-irregular for year i and month j. To simplify the calculations, only consider complete calendar years. That is, discard the observations from partly-recorded years at the start or finish of the data span, so that i runs from 1 to L, the number of complete years of data.

Let $\bar{X}_i = 1/12 \sum_{j=1}^{12} |SI_{ij} - 1.0|$ be the average of the absolute values of the mean-corrected seasonal-irregulars for each year,

$\bar{X}_j = 1/L \sum_{i=1}^{L} |SI_{ij} - 1.0|$ be the average of the absolute values of the mean-corrected seasonal-irregulars for each month, and

$\bar{X} = 1/n \sum_{i=1}^{L} \sum_{j=1}^{12} |SI_{ij} - 1.0|$, n = 12 * L be the average of the absolute values of all of the mean-corrected seasonal-irregulars.

Then total sum of squares

$$TSS = \sum_{i=1}^{L} \sum_{j=1}^{12} (|SI_{ij} - 1.0| - \bar{X})^2 ,$$

sum of squares due to years

$$SSY = 12 * \sum_{i=1}^{L} (\bar{X}_i - \bar{X})^2 ,$$

sum of squares due to months

$$SSM = L * \sum_{j=1}^{12} (\bar{X}_j - \bar{X})^2 , \text{ and}$$

sum of squares of error

SSE = TSS - SSY - SSM.

ANOVA Table

Source	Degrees of Freedom	Sum of squares	Mean squares
Years	L - 1	SSY	MSY = SSY/(L - 1)
Months	11	SSM	MSM = SSM/11
Error	n - L - 12 + 1	SSE	MSE = SSE/(n - L - 12 + 1)
Total	n - 1	TSS	

To test the null hypothesis that there is no difference between years, use the F statistic

F = MSY/MSE, and reject if $F > F\alpha$ based on $\nu 1 = (L - 1)$ and $\nu 2 = (n - L - 12 + 1)$ degrees of freedom.

If $F > F\alpha$ conclude that moving seasonality is present in the series, otherwise conclude that moving seasonality is not present.

The null hypothesis that there is no difference between months could also be tested using the F statistic F = MSM/MSE.
A one-way analysis of variance (without mean correction and without taking absolute values) may be more sensitive for testing for differences between months ie whether stable seasonality is present or not.

A one-way analysis of variance test for the presence of stable seasonality

The following calculations are for a test applied to monthly data. Calculations for quarterly data are very similar, the only difference is that in the quarterly case there are 4 periods per year rather than 12.

Let SI_{ij} be the multiplicative seasonal-irregular for year i and month j, and let n_j , j = 1,,12 be the number of observations in month j.

Let $\bar{X}_j = 1/n_j \sum_{i=1}^{n_j} SI_{ij}$ be the average of the seasonal-irregulars for each month, and

$\bar{X} = 1/n \sum_{i=1}^{n_j} \sum_{j=1}^{12} SI_{ij}$, $n = \sum_{j=1}^{12} n_j$ be the average of all of the irregulars.

Then total sum of squares

$$TSS = \sum_{i=1}^{N_j} \sum_{j=1}^{12} (SI_{ij} - \bar{X})^2,$$

sum of squares due to months

$$SSM = \sum_{j=1}^{12} n_j * (\bar{X}_j - \bar{X})^2, \text{ and}$$

sum of squares of error

SSE = TSS - SSM.

ANOVA Table

Source	Degrees of freedom	Sum of squares	Mean squares
Months	11	SSM	MSM = SSM/11
Error	n - 12	SSE	MSE = SSE/(n - 12)
Total	n - 1	TSS	

To test the null hypothesis that there is no difference between months, use the F statistic

F = MSM/MSE, and reject if $F > F\alpha$ based on $\nu 1 = 11$ and $\nu 2 = (n - 12)$ degrees of freedom.

If $F > F\alpha$ conclude that stable seasonality is present in the series, otherwise conclude that stable seasonality is not present.

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